ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : MAY / JUNE 2010

REGULATIONS : 2008

THIRD SEMESTER

080100008 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(COMMON TO AERONAUTICAL / AUTOMOBILE / BIOMEDICAL / CIVIL / CSE / IT / EEE / EIE / ECE / ICE / MECHANICAL / BIOTECH / CHEMICAL / FASHION TECH. / TEXTILE TECH. / TEXTILE CHEMISTRY)

TIME : 3 Hours **Max.Marks : 100**

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Define the root-mean square value of a function f(x) in $(0,2\pi)$.
2. State the Dirichlet's conditions for Fourier series.
3. If the half range – cosine series of $f(x)=x(\pi-x)$ in $(0,\pi)$ is given by $x(\pi-x)=\pi^2/6-\sum_{n=1}^{\infty}(1/n^2)\cos 2nx$, find the value of $1/1^4 + 1/2^4 + \ldots\ldots\ldots\ldots\infty$
4. What do you mean by Harmonic analysis.
5. State Fourier Integral theorem.
6. Find the Fourier sine transform of e^{-ax} (a>0).
7. State Parseval's identity for Fourier transform.
8. If $F\{f(x)\}=\overline{f(s)}$ then $F\{f(x)\cos ax\} = $ ---------------
9. Form the partial differential equation by eliminating the arbitrary function $z=f(x/y)$
10. Find the complete solution of the partial differential equation $\sqrt{p}+\sqrt{q}=1$
11. Find the particular integral of $(D^2+2DD'+D'^2)z=e^{x-y}$
12. Find the complete integral of the p.d.e. $z = px + qy + p^2 + q^2$

13. In the equation of motion of vibrating string $\frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, what does c^2 stand for?

14. What are the laws assumed to derive the one dimensional heat equation?

15. Write all the solutions of Laplace's equation $\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = 0$

16. If the ends of a string of length 'l' are fixed and the mid point of the string is drawn aside through a height 'h' and the string is released from rest, state the initial and boundary conditions.

17. Prove that $Z\left[(-1)^n\right] = \frac{z}{z+1}$

18. Define convolution of two sequences {f(n) }and {g(n)}.

19. Find the inverse Z– transform of $\frac{z}{(z-1)(z-2)}$

20. State initial value theorem in Z - transform.

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. (a) Find the Fourier Series of $f(x) = x + x^2$ in $(-\pi, \pi)$ of periodicity 2π 6

(b) Find the Fourier series expansion of period 2π for the function y = f(x) which is defined in (0, 2π) by means of the table of values given below. Find the series upto the second harmonic. 6

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

22. Find the Fourier transform of $f(x)=\begin{cases}1-x^2 in |x|\le 1\\ o \ in \ |x|>1\end{cases}$

Hence Prove that $\int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}ds=\frac{3\pi}{16}$

23. (a)Solve $(mz-ny)p+(nx-lz)q=ly-mx$ 6

(b)Solve $r+s-6t=y\cos x$ 6

24. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l-x)$, then show that $y(x,t)=\frac{8\lambda l^3}{\pi^4 a}\sum_{n=1,3,5}^{\infty}\frac{1}{n^4}\sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}$

25. (a) Find $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$ using convolution theorem. 6

(b) Solve the equation $y(n+2)-3y(n+1)+2y(n)=2^n$ given that y(0)=y(1)=0. 6

26. (a) Find the singular integral of $z=px+qy+\sqrt{p^2+q^2+1}$ 6

(b) Find $Z^{-1}\left[\frac{z^2-3z}{(z-5)(z+2)}\right]$ 6

27. An infinitely long plate in the form of an area is enclosed between the lines $y = 0$ and $y = \pi$ for positive value of x. The temperature is zero along the edges $y = 0$, $y = \pi$ and the edge at infinity. If the edge $x = 0$ is kept at the temperature $f(y) = ky$, $0 < y < \pi$, find the steady state temperature distribution in the Plate.

28. (a) Find the Fourier series for $f(x) = x^2$ in $(-\pi, \pi)$. Hence Show that 6

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\ldots = \frac{\pi^4}{90}$$

(b) Evaluate $\int_0^\infty \frac{dx}{(x^2+4)(x^2+1)}$ using Fourier transform method. 6

*****THE END*****